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Topic: Product Rule of Differentiation

### Product Rule of Differentiation

According to the product rule of derivatives, if the function  $f(x)$  is the product of two functions  $u(x)$  and  $v(x)$ , then the derivative of the function is given by:

If  $f(x) = u(x) \times v(x)$ , then:

$$f'(x) = u'(x) \times v(x) + u(x) \times v'(x)$$

**Example: Find the derivative of  $x^2(x+3)$ .**

Solution: As per the product rule of derivative, we know;

$$f'(x) = u'(x) \times v(x) + u(x) \times v'(x)$$

Here,

$$u(x) = x^2 \text{ and } v(x) = x+3$$

Therefore, on differentiating the given function, we get;

$$f'(x) = d/dx[x^2(x+3)]$$

$$f'(x) = d/dx(x^2)(x+3) + x^2 d/dx(x+3)$$

$$f'(x) = 2x(x+3) + x^2(1)$$

$$f'(x) = 2x^2 + 6x + x^2$$

$$f'(x) = 3x^2 + 6x$$

$$f'(x) = 3x(x+2)$$

### Quotient Rule of Differentiation

If  $f(x)$  is a function, which is equal to ratio of two functions  $u(x)$  and  $v(x)$  such that;

$$f(x) = u(x)/v(x)$$

Then, as per the quotient rule, the derivative of  $f(x)$  is given by;

$$f'(x) = \frac{u'(x) \times v(x) - u(x) \times v'(x)}{(v(x))^2}$$

**Example: Differentiate  $f(x) = (x+2)^3/\sqrt{x}$**

Solution: Given,

$$f(x) = (x+2)^3/\sqrt{x}$$

$$= (x+2)(x^2+4x+4)/\sqrt{x}$$

$$= [x^3+6x^2+12x+8]/x^{1/2}$$

$$= x^{-1/2}(x^3+6x^2+12x+8)$$

$$= x^{5/2}+6x^{3/2}+12x^{1/2}+8x^{-1/2}$$

Now, differentiating the given equation, we get;

$$f'(x) = 5/2x^{3/2} + 6(3/2x^{1/2})+12(1/2x^{-1/2})+8(-1/2x^{-3/2})$$

$$= 5/2x^{3/2} + 9x^{1/2} + 6x^{-1/2} - 4x^{-3/2}$$

## Chain Rule of Differentiation

If a function  $y = f(x) = g(u)$  and if  $u = h(x)$ , then the chain rule for differentiation is defined as;

$$\frac{dy}{dx} = \left(\frac{dy}{du}\right) \times \left(\frac{du}{dx}\right)$$

This rule is majorly used in the method of substitution where we can perform differentiation of composite functions.

Let's have a look at the examples given below for better understanding of the chain rule differentiation of functions.

### **Example 1:**

Differentiate  $f(x) = (x^4 - 1)^{50}$

### **Solution:**

Given,

$$f(x) = (x^4 - 1)^{50}$$

Let  $g(x) = x^4 - 1$  and  $n = 50$

$$u(t) = t^{50}$$

Thus,  $t = g(x) = x^4 - 1$

$$f(x) = u(g(x))$$

According to chain rule,

$$\frac{df}{dx} = \left(\frac{du}{dt}\right) \times \left(\frac{dt}{dx}\right)$$

Here,

$$\frac{du}{dt} = \frac{d}{dt} (t^{50}) = 50t^{49}$$

$$\frac{dt}{dx} = \frac{d}{dx} g(x)$$

$$= d/dx (x^4 - 1)$$

$$= 4x^3$$

$$\text{Thus, } df/dx = 50t^{49} \times (4x^3)$$

$$= 50(x^4 - 1)^{49} \times (4x^3)$$

$$= 200 x^3(x^4 - 1)^{49}$$

### **Example 2:**

Find the derivative of  $f(x) = e^{\sin(2x)}$

### **Solution:**

Given,

$$f(x) = e^{\sin(2x)}$$

Let  $t = g(x) = \sin 2x$  and  $u(t) = e^t$

According to chain rule,

$$df/dx = (du/dt) \times (dt/dx)$$

Here,

$$du/dt = d/dt (e^t) = e^t$$

$$dt/dx = d/dx g(x)$$

$$= d/dx (\sin 2x)$$

$$= 2 \cos 2x$$

$$\text{Thus, } df/dx = e^t \times 2 \cos 2x$$

$$= e^{\sin(2x)} \times 2 \cos 2x$$

$$= 2 \cos(2x) e^{\sin(2x)}$$